On Non-Smooth Non-Convex Non-Local Optimization

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Smooth Convex Optimization was the *paradigma* in Image Processing at the very beginning of this field through the celebrated Tikhonov Regularization for ill-posed inverse problems. It was in the 1990s when Perona and Malik, [1], introduced the necessity of nonlinearities in the flux of the diffusion equations. Soon after Rudin, Osher and Fatemi, [2] proposed a non-differentiable flux in the associated Euler-Lagrange Equation that was solved in a gradient descent approach for energy minimization. Non-smooth optimization emerged.

The Total Variation Operator and its famous counter-part in the Euler-Lagrange Equations, the $1-$ Laplacian Elliptic operator, surged ubiquitous in the Image Processing community. Since then an enormous quantity of models based on Bayesian modelling, energy minimization, elliptic and parabolic partial differential equations have appeared to cope with the (non exhaustive list of) image processing tasks of filtering, denoising, deblurring, deconvolution, segmentation, registration, impainting, superresolution.

To show the evolution of these ideas overcoming the *paradigma* of Convex and Local Optimization we shall consider hyper-Laplacian operators and related PDE to promote gradient sparse edge preserving solutions and non-local analysis.

In this talk I will present recent numerical results on some model problems that appear in Image Reconstruction as well as in more advanced tasks such as video sequences tracking and saliency for Human Activity Recognition. This presentation shall be based, essentially, on previous and current works of the author and his collaborators (see, for instance [3], [4], [5] and references therein).

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