Geometric Ergodicity of the Multivariate Continuous-time GARCH(1,1) Process

Stelzer Robert

(in collaboration with Johanna Vestweber)

Institute of Mathematical Finance, Ulm University, Ulm, Germany

In this talk we consider the multivariate continuous-time GARCH(1,1) process driven by a Lévy process emphasising stationarity properties. The focus is on the volatility process which takes values in the positive semi-definite matrices.

In the univariate model existence and uniqueness of the stationary distribution as well as geometric ergodicity are well-understood, whereas for the multivariate model only an existence criterion is known as far as strict stationarity is concerned. We shall first review the multivariate COGARCH(1,1) model and its properties focussing on strict and weak stationarity. Thereafter, the main part of the talk is devoted to establishing sufficient conditions for geometric ergodicity and thereby for uniqueness of the stationary distribution and exponential strong mixing.

We follow a classical Markov/Feller process approach based on a Foster-Lyapunov drift condition on the generator. Apart from finding an appropriate test function for the drift criterion, the main challenge is to prove an appropriate irreducibility condition due to the degenerate structure of the jumps of the volatility process, which are all rank one matrices. We present a sufficient condition for irreducibility in the case of the driving Lévy process being compound Poisson.

References

[1] Stelzer, R. Multivariate COGARCH(1,1) Processes, *Bernoulli*, **16**, 80–115, 2010.

[2] Stelzer, R., and Vestweber, J., Exponential Ergodicity of Multivariate COGARCH(1,1) Processes, *In Preparation*, 2016.