Graph limits of random graphs from a subset of connected k-trees

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For any set Ω of non-negative integers which contains $0, 1$ and at least one integer greater than 1, we consider a random Ω-k-tree $\mathsf{G}_{n,k}$ that is uniformly selected from the class of connected k-trees of $(n+k)$ vertices such that the number of $(k+1)$-cliques that contain any fixed k-clique belongs to the set Ω.

We establish the scaling limit and a local weak limit of this random Ω-k-tree $\mathsf{G}_{n,k}$. Since 1-trees are just trees, it is well-known that the random 1-tree with n vertices admits the Continuum Random Tree $\mathcal{T}_{\mathsf{e}}$ as the scaling limit and converges locally toward a modified Galton-Watson tree; see [1, 2, 3, 4]. We prove that the random Ω-k-tree $\mathsf{G}_{n,k}$, scaled by $(kH_k\sigma_\Omega)/(2\sqrt{n})$ where H_k is the k-th Harmonic number and σ_Ω is a positive constant, converges to the Continuum Random Tree $\mathcal{T}_{\mathsf{e}}$, too. In particular this shows that the diameter as well as the expected distance of two vertices in a random Ω-k-tree $\mathsf{G}_{n,k}$ are of order $\sqrt{n}$. Furthermore, we prove the local convergence of the random Ω-k-tree $\mathsf{G}_{n,k}$ to an infinite but locally finite random Ω-k-tree $\mathsf{G}_{\infty,k}$.

References

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