Minimal graphs containing k perfect matchings

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Let G be a graph. An *odd subdivision* of a graph G is obtained by replacing every edge of G with a path of odd length connecting its endvertices, so that these paths are internally disjoint. The replacement paths may also be of length one, making a graph an odd subdivision of itself.

Assuming G' is an odd subdivision of G there exists a natural bijective correspondence between perfect matchings in G and those in G'.

A graph G is *minimally k-matchable* if it has at least k distinct perfect matchings but deleting an arbitrary edge results in a graph which has fewer than k perfect matchings.

Let $k \geq 1$ be an integer. We show that there exists a *finite* set of graphs $\mathcal{G}_k$ so that every minimally k-matchable graph is isomorphic to a disjoint union of an odd subdivision of some graph from $\mathcal{G}_k$ and several copies of K_2.