Automorphism Groups of Planar Graphs and Spherical Groups

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By Frucht's Theorem, every abstract finite group is isomorphic to the automorphism group of some graph. In my talk I will consider the problem of characterisation of groups that acts as groups of symmetries of planar graphs. There are two important subfamilies of planar graphs. Namely, trees and polyhedral graphs. In 1869 Jordan gave an inductive characterisation of the automorphism groups of trees. The primitive groups are the symmetric groups, and operations used in the induction are direct and wreath products. As concerns the polyhedral graphs, their groups of symmetries are the well-known spherical groups. In 1975, Babai characterized the automorphism groups of planar graphs. However, it is not so easy to understand Babai's result. In the talk, we give a more detailed and understandable description of these groups following the approach by Jordan. We first describe stabilisers of vertices in the action of the automorphism groups of connected planar graphs as the class of groups closed under the direct product and certain semidirect products with symmetric, dihedral and cyclic groups. After that the automorphism group of a connected planar graph is obtained as a semidirect product of a direct product of these stabilizers with a spherical group. In order to understand the action of the spherical group on the complement we need to study in details the distribution of vertex- and edge-orbits in the action of a spherical group of a prescribed type on polyhedra. It seems that the best language to investigate it offers the concept of a map on the associated 2-dimensional quotient orbifold.