Holditch's theorem in space forms

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Consider a point on a chord of fixed length whose endpoints are moving on a closed convex planar curve. The locus of that point is a new closed curve. Holditch's theorem states that the area of the new curve is less than that of the original curve by $\pi\,p\,q$, being p and q the lengths of the pieces in which the chord is divided by the point.

A generalization of Holditch's theorem in space forms is studied, where it is possible to give a linear formula but now depending on the size of the original curve. This result was firsly obtained by Vidal Abascal and Rodeja (see [1] and [2]). Afterwards, Santaló gave a simpler proof of the same formula using differential forms, [3]. Our contribution is to give another proof focusing on the spherical and a hyperbolic version of the theorem without using differential forms and a to note a relation between the involved area and a quartic curve related to a kind of algebraic curves called cruciform curves.

References

[1] Abascal Vidal, E., Área engendrada sobre una superficie por un arco de geodésica cuando uno de sus extremos recorre una curva fija y longitud de la curva descrita por el otro extremo, *Revista Mat. Hisp.-Amer.* (4) **7** (1947), 132–142.

[2] Abascal Vidal, E. and E. G. Rodeja, Nota sobre curvas en superficies de curvatura constante, *Collectanea Math.* **num. V** (1952).

[3] Santaló, L. A., Sobre el teorema de Holditch y sus análogos en geometría no euclidiana, *Math. Notae* **14** (1954), 32–49.