

# Relative resolutions via truncations

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Our aim is to present a framework to do relative homological algebra. By this we mean that, if homological algebra is understood as a way to study objects in abelian categories through invariants determined by their injective resolutions, then we want to construct resolutions using as "injectives" a priori any class of objects. This idea we borrowed from homotopy theory, where the closely related idea of cellularization and  $A$ -homotopy theory (with an a priori space  $A$  and its suspensions taking the place usually devoted to the spheres) developed for instance by Farjoun has proved to be extremely fruitful. A convenient way to resolve unbounded complex  $X$  is to build a tower of left truncations of  $X$ , resolve each truncation in the usual way and then glue the resolutions back into a full resolution of  $X$ . The main problem arises in the "gluing back" process and is related to the fact that infinite products may fail to be exact. To solve this we will introduce a relative version of an extension of Grothendieck's axiom AB4\* due to Roos.