# Homogeneous geodesics and g.o. manifolds

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A geodesic in a homogeneous pseudo-Riemannian or affine manifold is homogeneous if it is an orbit of a 1-parameter group of isometries, or of affine diffeomorphisms. A homogeneous manifold is called a g.o. manifold if all geodesics are homogeneous.

Homogeneous geodesics were studied first in Riemannian manifolds. It is well known that in the symmetric space, all geoesics are homogeneous. A result by O. Kowalski and J. Szenthe says that in any homogeneous Riemannian manifold there exist at least one homogeneous geodesic through arbitrary point. Riemannian g.o. manifolds were studied by many authors and many examples were given. The interesting examples appear from dimension 6, because lower dimensional g.o. manifolds are naturally reductive. In pseudo-Riemannian manifolds, new phenomena appear. For example, for some light-like homogeneous geodesics, the natural parameter of the orbit is not the affine parameter of the geodesic. Further, the so-called almost g.o. manifolds appear. And the most interesting situation is with the nonreductive pseudo-Riemannian spaces, where the method used so far, and based on the reductive decomposition, fails. These spaces can be handled by the new, more fundamental, affine method, which was developed by the author, O. Kowalski and Z. Vlášek in [1]. The affine approach was also successfully applied in [2] and [3] to generalize the result on the existence of a homogeneous geodesic to pseudo-Riemannian and affine manifolds.

The present talk will be a survey on the interesting phenomena and examples related with this topic.

# References

[1] Z. Dušek, O. Kowalski, Z. Vlášek: Homogeneous geodesics in homogeneous affine manifolds, *Results in Math*, **54** (2009), 273-288.

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[3] Z. Dušek: The existence of light-like homogeneous geodesics in homogeneous Lorentzian manifolds, *Math. Nachr.*, **288**,8-9 (2015), 872-876.