Möbius inversion and bialgebras from decomposition spaces

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Decomposition spaces are simplicial infinity-groupoids that send generic (endpoint preserving) and free (distance preserving) pushout squares in the simplicial category Δ to (homotopy) pullbacks. They encode the information needed for an 'objective' generalisation of the notion of incidence (co)algebra of a poset, and turn out to coincide with the unital 2-Segal spaces of Dyckerhoff and Kapranov. Conditions for these coalgebras to have Möbius inversion can be given and a universal Möbius decomposition space can be constructed. Relevant constructions in many areas, in particular in combinatorics and algebraic topology, can be observed in this framework. Important examples of bialgebras in topology and theoretical physics arise from monoidal decomposition spaces.