A smooth Kerékjártó Theorem

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A continuous map $F : \mathbb{R}^n \to \mathbb{R}^n$ satisfying $F^m = \mathrm{Id}$ is called *m-periodic*. Here $F^j = F \circ F^{j-1}$ and m is the smallest positive natural number with this property. Usually, 2-periodic maps are called *involutions*.

Given $k = 0, 1, 2, \ldots, \infty$, we say that a map $F : \mathbb{R}^n \to \mathbb{R}^n$ of class $\mathcal{C}^k$ is (globally) *$\mathcal{C}^k$-linearizable* if it is conjugate to a linear map L via a $\mathcal{C}^k$-homeomorphism $\psi : \mathbb{R}^n \to \mathbb{R}^n$, that is, if $L = \psi \circ F \circ \psi^{-1}$. In dimension $n = 1$ it is not hard to prove that all periodic maps are $\mathcal{C}^0$-linearizable with $L(x) = x$ or $L(x) = -x$. A similar result holds for $n = 2$, now L is either the symmetry or a rotation of angle commensurable with 2π.

Kerékjártó Theorem. *Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a continuous m-periodic map. Then F is $\mathcal{C}^0$-linearizable.*

This result goes back to 1919 and appeared in the works of Brouwer and Kerékjártó. Currently it is known as Kerékjártó theorem. A complete proof was presented by Eilenberg in 1934 (see [2] for more details). In 1964 it was discovered by Bing that this theorem cannot be extended to higher dimensions.

In dimension $n = 1$, for any k, it is well-known that every non-trivial $\mathcal{C}^k$-periodic map is an involution and is $\mathcal{C}^k$-linearizable. In this talk we will give the key steps for proving the following $\mathcal{C}^k$ version of Kerékjártó theorem given in [1]:

Theorem. *Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a $\mathcal{C}^k$-differentiable m-periodic map with $k \in \{1, 2, \ldots, \infty\}$. Then F is $\mathcal{C}^k$-linearizable.*

References

[1] Cima, A., Gasull, A., Mañosas, F. and Ortega, R. Smooth linearization of planar periodic maps. Preprint 2015.

[2] Constantin, A. and Kolev, B. The theorem of Kerékjártó on periodic homeomorphisms of the disc and the sphere. *Enseign. Math.* **40**, 193–204, 1994.