Escaping points and semiconjugation of holomorphic self-maps of the punctured plane

David Martí-Pete

Department of Mathematics and Statistics, The Open University, Milton Keynes, United Kingdom

For every holomorphic self-map of the punctured plane f, there exists an entire funtion F that is semiconjugated to f by the exponential function - we say that F is a lift of f. Each holomorphic self-map of $\mathbb{C}^*$ has an associated index, $\text{ind}(f)$, which is an integer such that $F(z + 2\pi i) = F(z) + \text{ind}(f) \times 2\pi i$ for all $z \in \mathbb{C}$. We show that if f is a transcendental entire function with no zeros, then the fast escaping set of a lift F of f equals the preimage under the exponential of the fast escaping set of f. Bergweiler and Hinkkanen [1] proved one of the inclusions in a more general setting, but we show that equality holds in this particular case. Moreover, we can compare the escaping set, the set of unbounded non-escaping orbits and the set of bounded orbits of f with those of a lift F of f in terms of the index of f. Similar results hold for general holomorphic self-maps of $\mathbb{C}^*$.

References

[1] Bergweiler W., Hinkkanen, A. On semiconjugation of entire functions *Math. Proc. Cambridge Philos. Soc.*, **126**, no. 3, 565-574, 1999.