

Estimates for order of Nevanlinna matrices

RAPHAEL PRUCKNER

(in collaboration with Harald Woracek)

Vienna University of Technology

All solutions of an indeterminate Hamburger moment problem can be described using a Nevanlinna matrix. The entries of this matrix are entire functions of order less or equal 1. The problem is to determine the exact order of these functions.

We write the moment problem as a canonical system with Hamiltonian H . Here, $H : [0, L) \rightarrow \mathbb{R}^{2 \times 2}$ is a locally integrable function whose values are a.e. positive semi-definite. The corresponding canonical system is given by the equation

$$y'(x) = zJH(x)y(x), \quad x \in (0, L),$$

where $z \in \mathbb{C}$ and $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The so-called Hamburger Hamiltonians which appear here have a much easier structure.

We obtain estimates for the order by transforming a given Hamburger Hamiltonian into (the Hamiltonian associated with) a Krein-string, and apply a theorem of I.S.Kac to evaluate the order of that string. Our result can be viewed as a generalisation of a theorem by Berezanskii in the 50s.

On the way, we leave the positive definite scheme and encounter Hamiltonians which take also negative definite matrices as values.