Diffusion-type equations on discrete-space domains

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We focus on diffusion-type equations of the form

$$u^{\Delta}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t), \quad x \in \mathbb{Z},\ t \in \mathbb{T}, \tag{1}$$

where $\mathbb{T}$ is an arbitrary time scale (i.e., a closed subset of $\mathbb{R}$), u^{Δ} denotes the Δ-derivative of u with respect to t, and a, b, $c \in \mathbb{R}$.

When $\mathbb{T} = \mathbb{R}$, $u^{\Delta}(x,t)$ becomes the usual partial derivative $u_t(x,t)$, and Eq. (1) generalizes the space-discretized version of the classical diffusion equation. For $\mathbb{T} = \mathbb{Z}$, $u^{\Delta}(x,t)$ reduces to the partial difference $u(x,t+1) - u(x,t)$, and Eq. (1) describes the one-dimensional (not necessarily symmetric) random walk on $\mathbb{Z}$.

We study the existence and (non)uniqueness of solutions to initial-value problems, superposition principle, space sum preservation, and maximum and minimum principles.

Eq. (1) can be generalized in various ways. For example, the spatial domain can be $\mathbb{Z}^n$ or a general graph. Another possibility is to consider nonlinear reaction-diffusion equations of the form

$$u^{\Delta}(x,t) = au(x+1,t) + bu(x,t) + cu(x-1,t) + f(u(x,t),x,t), \quad x \in \mathbb{Z},\ t \in \mathbb{T}. \tag{2}$$

Special cases of Eq. (2) include the Fisher and Nagumo lattice equations, or nonautonomous logistic population models with a variable carrying capacity.

References

[1] A. Slavík, P. Stehlík, *Dynamic diffusion-type equations on discrete-space domains*, J. Math. Anal. Appl. **427** (2015), no. 1, 525–545.

[2] A. Slavík, P. Stehlík, and J. Volek, *Well-posedness and maximum principles for lattice reaction-diffusion equations*, submitted for publication.